Gluts in the logic of anaphoric dependencies

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https://patrickdelliott.com/pdf/silt.pdf

- *Donkey sentences* oscillate between ∃/∀-readings, subject to contextual factors (Kanazawa 1994, Barker 1996, Champollion, Ciardelli & Zhang 2016 among many others).
- My empirical contribution: the ∃/∀-ambiguity persists in cases of cross-sentential anaphora, focusing on anaphora in *disjunction*.
- Ultimately, I'll suggest that the ∃/∀-ambiguity is a property of anaphoric dependencies for a principled reason:
  - An logic of anaphoric accessibility which validates certain equivalences invariably gives rise to *truth value gluts* - cases where ϕ and $\neg\phi$ are *compatible*.
  - Gluts allow for ϕ to be interpreted *exhaustively* relative to $\neg\phi$, subject to relevance; exhaustive interpretation is responsible for selectively strengthening the truth-conditions of donkey sentences.

- Background: dynamics of anaphora in complex sentences and ∃/∀-readings.
- Donkey disjunctions: motivating ∃/∀-readings in disjunctive sentences.
- Bilateral Update Semantics: a framework for reasoning about anaphoric accessibility.
- Gluts and exhaustive interpretation
- Conclusion
- Appendix: ∀-readings of discourse anaphora.

Background

(1) If a farmer owns a donkey, he treasures it.

(2) Every farmer who owns a donkey treasures it.

(3) Most farmers who own a donkey treasure it.

- An indefinite in the restrictor of a quantificational expression *Q* can bind a pronoun in *Q*'s scope.
- Famously resists treatment as (in-scope) variable binding; a primary empirical motivation for *dynamic semantics* (Heim 1982, Kamp 1981, Groenendijk & Stokhof 1991).

(4) Giles owns a donkey. He treasures it.

(5) Giles owns a donkey and he treasures it.

- Another kind of anaphora that resists classical treatments.
- More motivation for dynamic semantics.

(6) Either Giles doesn't own a donkey, or he treasures it.

(7) It's not true that Giles doesn't own a donkey! He treasures it.

- Dynamic approach to donkey/discourse anaphora can be extended to account for these cases too (Krahmer & Muskens 1995, Gotham 2019, Mandelkern 2022, Elliott 2020, Hofmann 2022).

- First-generation dynamic approaches such as Groenendijk & Stokhof's *Dynamic Predicate Logic* (DPL) made the following predictions for discourse vs. donkey anaphora.
  - Discourse anaphora gives rise to *existential truth-conditions*, thanks to Egli's theorem: $\exists_x \phi \wedge \psi \iff \exists_x (\phi \wedge \psi)$
  - Donkey anaphora gives rise to *universal truth-conditions*, thanks to Egli's corrolary: $\exists_x \phi \rightarrow \psi \iff \forall_x (\phi \rightarrow \psi)$
  - We'll discuss the truth conditions of Partee disjunctions later.

(8) Giles owns a donkey and he treasures it.

$\iff$ *Some donkey is owned and treasured by Giles.*

(9) If Giles owns a donkey he treasures it.

$\iff$ *Every donkey owned by Giles is treasured by him.*

In later years, a more nuanced picture emerged: donkey pronouns can be associated with universal *or* existential force (Chierchia 1995, Kanazawa 1994, 2001).

(10) Every farmer who owns a donkey feeds it well.
$\Rightarrow$ *Every farmer who owns a donkey feeds each of his donkeys well.*

(11) No farmer who owns a donkey feeds it well.
$\Rightarrow$ *No farmer who owns a donkey feed any of his donkeys well.*

Although factors such as monotonicity clearly play a role (Kanazawa 1994), context is also important - universal quantifiers are for example sometimes compatible with existential readings.

(12) *Q: Did anyone get wet?*
Everyone who has an umbrella remembered to bring it with them.
$\Longleftrightarrow$ *Everyone who has an umbrella remembered to bring one of their umbrellas*.

This is a heterogeneous reading (Champollion, Bumford & Henderson 2019) , since it is true in a *mixed* scenario, where some may also have an umbrella they didn't bring; previous examples are homogeneous.

The state of the art: truth-value gaps

Champollion, Bumford & Henderson (2019) develop an account of donkey anaphora, drawing inspiration from work on homogeneity and plural definites (Križ 2017).

Concretely, they claim that donkey sentences give rise to truth-value gaps.

(13) Every farmer who owns a donkey feeds it well.

a. True if *every donkey owning farmer feeds all of his donkeys well.*

b. False if *At least one donkey-owning farmer doesn't feed any of his donkeys well.*

c. Undefined *otherwise*

Together with a pragmatic bridge principle, CBH's approach can capture both homogeneous and heterogeneous readings, but there are some issues.

- Limited empirical remit: CBH's account is tailored specifically for quantificational environments. I'll suggest that the ∃/∀ distinction is more general thant this; the key evidence will come from *Partee disjunctions*.
- Requires non-uniform bridge principle: their proposal requires an idiosyncratic treatment for sentences with free pronouns (p. 22).

An alternative: glutty truth-conditions

I'll argue that the source of ∃/∀-readings rather is rather lies in the fact that donkey sentences (among others) give rise to *truth-value gluts*.

(14) Every farmer who owns a donkey feeds it well.

a. True if *every donkey owning farmer has a donkey that he feeds well*

b. False if *At least one donkey-owning farmer has a donkey that he doesn't feed well*

- Note that in a *mixed* scenario, this sentence is *both true and false*.
- Of course, this is to be avoided! The basic idea will be that to derive homogeneous readings, donkey sentences are generlly assumed to be *just true* (i.e., true and not false).

- The main empirical motivation for the glutty theory will come from ∀/∃-readings in non-quantificational environments, i.e., with discourse anaphora in disjunctive sentences.
- It will turn out that the presence of gluts will be a natural consequence of a principled logic of anaphora which extends to discourse anaphora in disjunctions.

Donkey disjunctions

Discourse anaphora is typically blocked by disjunction (Groenendijk & Stokhof 1991).

(15) # Either Matthew has a smart shirt, or it's in his closet.

Barbara Partee however famously observed that anaphora is possible if a witness to an indefinite in an initial disjunct is entailed by its *negation*.

(16) Either Matthew doesn't have a smart shirt, or it's in his closet.

Krahmer & Muskens (1995) claim that the following Partee disjunction is false in the provided context (p. 362):

(17) Context: *there are two bathroom in this house; one of them is in a strange place, and one isn't.*
Either there's no bathroom in this house,
or it's in a strange place.

This is to be expected if Partee disjunctions have *universal* truth conditions:

$$\forall_x(\mathbf{Bathroom}(x) \rightarrow \mathbf{Strange}(x))$$

Elliott (2022) shows that, given the right context, Partee disjunctions can however have an existential reading.

(18) Context: *We're wondering whether Gabe paid with cash or card.*
Either Gabe doesn't have a credit card with him,
or he paid with it.

- *False* if Gabe has credit cards with him, and didn't pay with *any of them*.
- *True* if Gabe has credit cards and paid with at least one of them.
- Elliott's proposed truth-conditions:

$$\neg\exists_x \mathbf{Card}(x) \vee \exists_x(\mathbf{Card}(x) \wedge \mathbf{Pay}(x))$$

Existing accounts: an impasse

- Krahmer & Muskens (1995) develop a variant of DRT tailored to derive only *universal* readings.
- Elliott (2022) develops a trivalent dynamic semantics tailored to derive only *existential* readings.
  - Mandelkern (2022), Hofmann (2019, 2022) don't explicitly discuss the truth-conditions of Partee disjunctions, but seem to also derive existential truth conditions(?).
- There are also accounts which claim that Partee disjunctions entail *uniqueness* (Gotham 2019), but sage-plant sentences show putative uniqueness inferences are cancellable (Mandelkern & Rothschild 2020).

In essence, (Krahmer & Muskens 1995) and (Elliott 2022) differ in which of the following (classically equivalent!) formulas captures Partee disjunctions.

(19) $\neg\phi \vee (\phi \to \psi)$

(20) $\neg\phi \vee (\phi \wedge \psi)$

- Egli's theorem/corrolary mean that they give rise to different truth-conditions, if an existential in ϕ binds a pronoun in ψ.
- In the following, I show that context can militate between ∀- and ∃-readings.

(21) Context: *spoken by a donkey welfare activist organizing boycotts against farmers who aren't kind to their donkeys:*
Every farmer who doesn't have a donkey or feeds it treats is safe from reprisal.

The *donkey welfare activist* is insinuating that any farmer who neglects any of his donkeys is subject to reprisal.

$$\forall_f(\overbrace{(\neg\exists_d \mathbf{Own}(f,d) \vee \forall_d(\mathbf{Own}(f,d) \rightarrow \mathbf{Treats}(f,d)))}^{\text{Partee disjunction}} \rightarrow \mathbf{Safe}(f))$$

(22) Context: *spoken by a cruel industrialist who believes that starving donkeys are the most efficient workers.*
Every farmer who doesn't have a donkey or feeds it treats is working below maximum efficiency.

The *cruel industrialist* is insinuating that a farmer who indulges *any* of his donkeys is working below maximum efficiency.

$$\forall_f(\overbrace{(\neg\exists_d \mathbf{Own}(f,d) \vee \exists_d(\mathbf{Own}(f,d) \wedge \mathbf{Indulge}(f,d))}^{\text{Partee disjunction}} \rightarrow \mathbf{BelowMax}(f))$$

Assume we have a theory that derives existential truth conditions for Partee disjunctions:

(23) $\neg\exists_x P(x) \vee Q(x)$
…true iff there is no x s.t. $P(x)$ is true
or there is some x s.t. $P(x)$ is true and $Q(x)$ is true.

By de Morgan's, and double-negation elimination, a *negated* Partee disjunction is equivalent to discourse anaphora, which standardly gets existential truth conditions:

(24) $\neg(\neg\exists_x P(x) \vee Q(x))$

$$\iff \neg\neg\exists_x P(x) \wedge \neg Q(x)$$

$$\iff \exists_x P(x) \wedge \neg Q(x)$$

…true iff there is some x s.t. $P(x)$ is true and $Q(x)$ is false.

- Concretely, consider a mixed scenario, where there is some x s.t. $P(x)$ is true and $Q(x)$ is true, and there is some y s.t., $P(y)$ is true, and $Q(y)$ is false.
  - $\neg\exists_x P(x) \vee Q(x)$ is true
    (by existential truth-conditions for Partee disjunctions)
  - $\neg(\exists_x P(x) \vee Q(x))$ is true
    (by de Morgan's, double-negation elimination, and existential truth-conditions for discourse anaphora)
- This is a kind of glut; in a mixed scenario we have a sentence ϕ, s.t., ϕ and $\neg\phi$ are both true.
- In certain contexts, ϕ is interpreted exhaustively relative to $\neg\phi$.

Bilateral Update Semantics

- In previous work, Elliott (2020, 2022, 2023) has developed an account of anaphoric accessibility based on a *predictive* theory of presupposition projection by embedded Strong Kleene trivalent semantics in a dynamic setting.
    - There are two variations: Existential Dynamic Semantics (Elliott 2023) and Bilteral Update Semantics (Elliott 2022); I use the latter here for expository reasons.

- An expression ϕ is associated with:
    - A positive update $[\phi]^+$.
    - A negative update $[\phi]^-$.
- Updates are functions from Heimian information states to information states.
- The positive update $c[\phi]^+$ often (but not always) corresponds to the effect of asserting ϕ against a context set c.

(25) $s[\text{it}_x \text{ is upstairs}]^+ := \{ (w, g) \in s \mid g_x \text{ is upstairs}_w \}$

(26) $s[\text{it}_x \text{ is upstairs}]^- := \{ (w, g) \in s \mid g_x \text{ isn't upstairs}_w \}$

- Atomic sentences are associated with a positive/negative update which picks out the possibilities in s at which the sentence is true/false respectively.
  - We assume that assignments are partial, which means that $s[\phi]^{+,-}$ doesn't always partition s.
  - In order to capture Heimian familiarity, we assume that $c[\phi]^{+,-}$ must partition c in order for ϕ to be assertable at c.

The positive update of an existential statement introduces a discourse referent, just like in ordinary update semantics.

(27) $s[\text{there is a}^x \text{ bathroom}]^+ :=$
$\{(w,h) \mid (w,g) \in s, g[x]h \wedge h_x \text{ bathroom}_w\}$

Crucially, the negative update of an existential statement simply picks out possibilities in s at which there is no bathroom, without introducing any anaphoric information (Mandelkern 2022).

(28) $s[\text{there is a}^x \text{ bathroom}]^- :=$
$\{(w,g) \in s \mid \text{there is no bathroom in } w\}$

DNE is validated by a standard (flip-flop) entry for negation:

(29) $s[\text{not } \phi]^{+} := s[\phi]^{-}$

(30) $s[\text{not } \phi]^{-} := s[\phi]^{+}$

- It's obvious that this entry validates DNE, since $s[\neg\neg\phi]^{+} = s[\neg\phi]^{-} = s[\phi]^{+}$, and $s[\neg\neg\phi]^{-} = s[\neg\phi]^{+} = s[\phi]^{-}$.
  - This means that, e.g., $s[\text{there's no}^{x}\text{bathroom}]^{-}$ will introduce a bathroom discourse referent. This will be crucial for our account of Partee disjunctions.

In BUS, we cash out the Strong Kleene truth table as a recipe for constructing positive/negative updates of complex expressions.

$\phi \vee \psi$	ψ_+	ψ_-	$\psi_?$
ϕ_+	+	+	+
ϕ_-	+	−	?
$\phi_?$	+	?	?

Figure 1: Strong Kleene disjunction

Each +, – cell is interpreted as an instruction to perform a successive update. In order to get the result of the positive update of $s[\phi \vee \psi]^+$, we take the union of all of the successive updates represented by the + cells.

The unknown update corresponds to the unknown truth-value in Strong Kleene trivalent logic.

(31) $s[\phi]^{?} = \{\, i \in s \mid i \not\prec s[\phi]^{+,-} \,\}$

- The *unknown update* of s by ϕ picks out the possibilities in s which don't *subsist* (Groenendijk, Stokhof & Veltman 1996) in either the positive or negative update.
- The simplest case: $s[\phi]^{?}$ picks out the possibilities in s which are neither in the positive, nor the negative update.

(32) $s[\text{it}_x\text{'s upstairs}]^{?} := \{\, (w, g) \in s \mid g_x \text{ is undefined} \,\}$

We can think of the bridge principle as the requirement that $s[\phi]^{?} = \varnothing$

By the Strong Kleene truth-table, we must compute the following:

(33) $$s[\phi \vee \psi]^+ := s[\phi]^+[\psi]^+ \cup s[\phi]^+[\psi]^- \cup s[\phi]^+[\psi]^? \cup s[\phi]^-[\psi]^+ \cup s[\phi]^?[\psi]^+$$

- The first line corresponds to dynamically verifying the disjunction by the truth of the first disjunct.
- The second line corresponds to dynamically verifying the disjunction by the truth of the second disjunct.

If we assume that the first disjunct is true, the contribution of the second disjunct is trivial (since it doesn't introduce any discourse referents).

(34) s[there is no bathroom]$^+$[it's upstairs]$^{+,-,?}$ =
s[there is no bathroom]$^+$

- Generally, if ϕ is atomic, then $s[\phi]^+ \cup s[\phi]^- \cup s[\phi]^? = s$

- If the first disjunct is *false*, then by DNE it will introduce a DR.
  - Verifying the disjunction in this case depends on the second disjunct being true.

(35) s[there is no bathroom]$^-$[it's upstairs]$^+$ =
s[there's a bathroom]$^+$[it's upstairs]$^+$

- The $s[\phi]^?[\psi]^+$ case is irrelevant, since the first disjunct (an existential statement) is bivalent.

The positive update associated with the Partee disjunction is the union of the updates associated with (i) veryifing via the first disjunct, and (ii) verifying via the second.

$$s[\text{either there's no bathroom}^{x} \text{ or it}_x\text{'s upstairs}]^{+}$$
$$= \{(w, g) \in s \mid \text{no bathroom in } w\}$$
$$\cup \{(w, h) \mid (w, g) \in s, g[x]h, h_x \text{ an upstairs bathroom in } w\}$$

Possibilities where no bathrooms exist are retained, and bathroom-upstairs possibilities are associated with a bathroom discourse referent.

The negative case is much simpler, since the only way of dynamically falisfying a disjunction is if both disjuncts are false. This amounts to discourse anaphora.

(36) Either there's no^x bathroom, or it$_x$'s upstairs.

(37) s[either there's no bathroom^x or it$_x$'s upstairs]$^- =$

$$\left\{ (w,h) \;\middle|\; \begin{array}{l} (w,g) \in s, g[x]h \\ \wedge\, h_x \text{ is a non-upstairs bathroom in } w \end{array} \right\}$$

- Note, de Morgan's equivalences and DNE go through, so $\neg(\neg\exists_x B(x) \vee U(x)) \iff \exists_x B(x) \wedge \neg U(x)$.
  - Negated Partee disjunctions have a *weak, existential* reading.

Gluts and exhaustive interpretation

- Partee disjunctions aren't *gappy*, since $s[.]^?$ is empty.
  - Non-bathroom worlds are in $s[.]^+$
  - Bathroom upstairs worlds are in $s[.]^+$
  - Bathroom downstairs worlds are in $s[.]^-$
- Partee disjunctions are glutty: $s[.]^+$ and $s[.]^-$ aren't disjoint.
  - *Mixed* worlds, in which there is, e.g., a bathroom upstairs and a bathroom downstairs, count as both *bathroom upstairs world*, and *bathroom downstairs* worlds, and therefore are in both $s[.]^+$ and $s[.]^-$.
- In general, this isn't hugely problematic: we might assume a bridge principle that simply outputs $s[.]^+$, as long as $s[.]^?$ is empty; $s[.]^- - s[.]^+$ is discarded.
  - However, this only generates ∃-readings both for Partee disjunctions and their negated counterparts.

- First, we'll show how to derive homogeneous readings, via a pragmatic principle.
    - Our initial attempt will however *only* derive homogeneous readings.
- Subsequently, we'll show how to account for the availability of hetereogeneous readings given the right context, by relativizing the pragmatic principle to a QuD (Roberts 2012, Champollion, Bumford & Henderson 2019).

Polar questions can be understood induce an equivalence relation between worlds (Groenendijk & Stokhof 1984).

(38) Is it raining?

(39) $\lambda w\,.\,\lambda w'\,.$ it's raining in $w \iff$ it's raining in w'

- Cell 1: *raining possibilities.*
- Cell 2: *Not-raining possibilities.*

In the analysis of *wh*-questions, it's common to *derive* this equivalence relation from a set of alternatives (Heim 1994).

(40) $w \sim w' \iff \forall p \in Q[p(w) = p(w')]$

In the case of polar questions, we can think of the alternatives in Q as corresponding to the *yes*, and *no* answers to the question.

(41) { it's raining, it's not raining }

This of course gives rise to the same two cells.

Now consider how we might derive an equivalence relation from the following alternatives:

$$\left\{\begin{array}{l}\text{either there's no bathroom or it's upstairs,}\\ \text{neither is there no bathroom, nor is it upstairs}\end{array}\right\}$$

Equivalently:

$$\left\{\begin{array}{l}\exists_x B(x) \rightarrow \exists_x (B(x) \wedge U(x)),\\ \exists_x (B(x) \wedge \neg U(x))\end{array}\right\}$$

This gives rise to *three* cells(!); *yes* and *no* answers identify cells 1 and 2 respectively.

- Cell 1: contains possibilities where every bathroom (if any) is upstairs.
  - $\exists_x B(x) \rightarrow \exists_x (B(x) \wedge U(x)) \wedge \neg \exists_x (B(x) \wedge \neg U(x))$
  - $\Rightarrow \forall_x (B(x) \rightarrow U(x))$
- Cell 2: contains possibilities where every bathroom (if any) is downstairs.
  - $\exists_x (B(x) \wedge \neg U(x)) \wedge \neg(\exists_x B(x) \rightarrow \exists_x (B(x) \wedge U(x)))$
  - $\Rightarrow \forall_x (B(x) \rightarrow \neg U(x))$
- Cell 3: contains possibilities where at least one bathroom is upstairs, and at least one bathroom is downstairs.
  - $\exists_x (B(x) \rightarrow_x (B(x) \wedge U(x)) \wedge \exists_x (B(x) \wedge \neg U(x))$
  - $\Rightarrow \exists_x, y(B(x) \wedge B(y) \wedge U(x) \wedge \neg U(y))$

- The intuition behind our pragmatic principle is that a sentence ϕ, when asserted, is (by default) interpreted *exhaustively* relative to the polar question induced by $\{\phi, \neg\phi\}$.
- In a classical setting, this is trivial - if ϕ is true, then $\neg\phi$ is false, so ϕ is used to uniquely identify the ϕ cell.
- In a *glutty* setting, where ϕ and $\neg\phi$ can both be true, then interpreting ϕ exhaustively amounts to using ϕ to identify the cell in which ϕ is true and $\neg\phi$ isn't.

- For expository purposes, we can build exhaustive interpretation directly into our bridge principle as follows; sentences are assumed to be "just true".

(42) Bridge (v1): ϕ asserted at c results in
$\{\,(w, g) \in c[\phi]^{+} \mid (w, *) \notin c[\phi]^{-}\,\}$ if $c[\phi]^{?} = \varnothing$ else $\varnothing$

- It's easy to see what the consequences are relative to a Partee disjunction such as "Either there is no bathroom, or it's upstairs":
  - In the positive case, assertion only leaves possibilities in which every bathroom (if any) is upstairs (the homogeneous reading).
  - In the negative case, assertion only leaves possibilities in which there is a bathroom, and every bathroom is downstairs (again, the homogeneous reading).

As we've seen, we *don't* want to have homogeneous readings everywhere, so we need a way of weakening the predictions of exhaustive interpretation, relative to contextual factors.

Here, we use the same technique as (Križ 2017, Champollion, Bumford & Henderson 2019) and model the context as a salient *Question under Discussion* (QuD) (Roberts 2012).

We assume that assertion is always made relative to a (possibly implicit) question, which is modelled as a contextually salient equivalence relation $\sim_Q$.

(43) Bridge (final): ϕ asserted at c results in $\{\, (w, g) \in c[\phi]^+ \mid \exists w', w \sim_Q w', (w', *) \notin c[\phi]^- \,\}$ if $c[\phi]^? = \varnothing$ else $\varnothing$

- Relative to a maximally specific *fact-finding* questions, where no two worlds are Q-equivalent, the QuD makes no difference and homogeneous readings are derived.
  - Heterogeneous readings are derived by taking a broader QuD, as we'll show in more detail in the following.

- Imagine a context where the QuD is: *how did Gennaro pay for dinner*.
    - The context is divided into cells according to how Gennaro paid (i) a cell containing worlds in which Gennaro paid with a credit card, and in no other way, (ii) a cell containing worlds in which Gennaro paid with cash, and in no other way.
    - Note that the question doesn't draw any distinctions according to how many credit cards Gennaro has.

Now consider an assertion of the following sentence, in light of its negative contribution:

(44) Either Gennaro doesn’t have a credit card, or he paid with it.

(45) Neither does Gennaro have a credit card, nor did he pay with it.
⇒ *Gennaro has a credit card that he didn’t pay with*

- Assertion will leave possibilities in which Gennaro has multiple credit cards and just paid with one of them, so long as it’s contextually possible that there is no credit card which wasn’t used for paying.
    - Since the modified bridge makes these possibilities Q-equivalent, the hereogeneous reading is derived.

(46) Context: *Did Josie get wet? I bet she either doesn't have an umbrella, or left it at home.*
Neither does Josie have no umbrella, nor did she leave it at home!

- If the QuD distinguishes worlds in which Josie got wet, from worlds in which she didn't.
    - The distinction between worlds in which Josie brought one her umbrellas, and left the others at home, and worlds in which she brought all her umbrellas is elided.
    - What's relevant is whether she left *all of her umbrellas* at home.
    - This is a *heteogeneous* ∀*-reading*.

In BUS, unlike in classical dynamic theories such as DPL, FCS (Groenendijk & Stokhof 1991, Heim 1982), many classical equivalences go through, e.g.:

(47) $\phi \rightarrow \psi \iff \neg\phi \vee \psi$

(48) If there's a bathroom then it's upstairs.

(49) Either there isn't a bathroom or it's upstairs.

- The account of ∃/∀-readings in disjunctions therefore straightforwardly carries over to conditionals via material implication.
  - This is however an unrealistic analysis for natural language conditionals. In future research, I plan to extend the glutty analysis to quantificational environments, including both conditionals and generalized quantifiers.

Conclusion

- One striking (and on the face of it, wrong) prediction is that *discourse anaphora* should give rise to ∃/∀-readings. This is briefly discussed in the appendix, reporting results from (Chatain 2018).
- Another clear question is whether there are other glutty environments in natural language.
  - One intriguing possibility is to analyze sentences with definite plurals and their negations as having glutty, existential truth conditons:

(50) The boys left. *at least one boy left.*

(51) The boys didn't leave. *at least one boy didn't leave.*

- I’ve shown that the famous ∃/∀ ambiguity observed in donkey sentences is a more general phenomenon; it can be detected in other cases of cross-sentential anaphora.
- This suggests that a more general account is called for. We showed that any logical theory of accessibility with the following properties gives rise to *gluts*:
  - Derives existential truth conditions for Partee disjunctions.
  - Validates de Morgan’s equivalences.
- A theory which gives rise to gluts automatically gives rise to new possibilities wrt exhaustive interpretation.

$\mathcal{F}in$

Appendix: universal readings of discourse anaphora

- In the final section, I'll discuss a very surprising consequence for discourse anaphora.
  - It can be demonstrated that standard discourse anaphora in conjunctions gives rise to gluts.
  - We therefore expect it to give rise to both ∃- and ∀-readings.
  - The ∀-reading is difficult to detect, but evidence nevertheless suggests that it is available.

Egli's theorem isn't valid in BUS, but a weaker equivalence nevertheless holds:

$$[\exists_x P(x) \land Q(x)]^+ = [\exists_x (P(x) \land Q(x))]^+$$

In the negative case, negated discourse anaphora can conditionally introduce a discourse referent, if the conjunction is falsified by the second conjunct; a negated exisential never introduces a discourse referent.

$$[\exists_x P(x) \land Q(x)]^- \neq [\exists_x (P(x) \land Q(x))]^-$$

A different way of showing this is by observing that de Morgan’s equivalence can be applied to discourse anaphora to derive a Partee disjunction.

(52) Giles owns a donkey and he beats it.
True iff *Giles owns a donkey, and he beats a donkey he owns*

(53) It’s not true that Giles owns a donkey and he beats it.
True iff *Giles doesn’t own a donkey, or he owns a donkey that he doesn’t beat*

- In a mixed scenario, where Giles owns one donkey he beats, and one that he doesn’t, the sentence and its negation are both true.
- By the logic of exhaustive interpretation, we therefore expect discourse anaphora to (potentially) receive a ∀-reading.

- Chatain (2018) has provided some evidence that discourse anaphora can, counter-intuitively, sometimes have universal truth-conditions.
- Chatain formulates the following examples (from p.182) as *bets*, in an attempt to probe unstrengthened truth-conditions (Schlenker 2016).
  - This is important, since *uniqueness inferences* associated with singular indefinites otherwise collapse the $\forall/\exists$ distinction.

Chatain reports - importantly for our purposes - that a subset of informants judge that the speaker loses her bet if Camelia has any of her umbrellas with her.

(54) I bet you 10 bucks that Camelia has an umbrella and that she left it at home today.

a. ∀-reading: ...*Camelia has an umbrella and she left all of her umbrellas at home today.*

   Prediction: bet lost iff Camelia has any of her umbrellas with her.

b. ∃-reading: ...*Camelia has an umbrella and she left one of her umbrellas at home today.*

   Prediction: bet lost iff Camelia has all of her umbrellas with her.

It's important to compare with a baseline involving an indefinite taking scope over a conjunction:

(55) *Speaker is arguing that Camelia is absent-minded, hearer disagrees.*

a. I bet you 10 bucks that there is an umbrella belonging to Camelia that she left at home today.

b. **Outcome:** Camelia has 10 umbrellas. She brought one, and left the others.

Uncontroversially here, the bet is won, as we expect on the basis of existential truth-conditions.

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