Finding strength in contradiction

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https://patrickdelliott.com/pdf/pallmyr.pdf

- *Donkey sentences* oscillate between so-called ∃- and ∀-readings, subject to contextual factors (Kanazawa 1994, Chierchia 1995, Barker 1996, Champollion, Ciardelli & Zhang 2016 a.o.).
- My empirical contribution: the ∃/∀ ambiguity persists in cases of cross-sentential anaphora more generally - I focus on anaphora in disjunctions.
- Ultimately, I'll suggest that the ∃/∀ ambiguity arises for principled reasons:
  - A logic of anaphoric accessibility which validates certain natural equivalences gives rise to cases where ϕ and $\neg\phi$ are *compatible*.
  - This property allows for ϕ to be interpreted *exhaustively* relative to $\neg\phi$; exhaustive interpretation is responsible for selectively strengthening the truth-conditions of donkey sentences.

Roadmap

- Background: dynamics of anaphora in complex sentences, and $\exists/\forall$-readings.
- Bathroom disjunctions: motivating $\exists/\forall$-readings in disjunctive sentences.
- Existential Dynamic Semantics: a framework for reasoning about anaphoric accessibility.
- Homogeneous readings: deriving homogeneous readings by strengthening ϕ relative to $\neg\phi$.
- Conclusion

Background

(1) If a farmer owns a donkey, he treasures it.

(2) Every farmer who owns a donkey treasures it.

(3) Most farmers who own a donkey treasure it.

- Descriptively: an indefinite in the restrictor of a quantificational expression *Q* can bind a pronoun in *Q*'s scope.
- - Famously resists treatment as (in-scope) variable binding; a primary empirical motivation for *dynamic semantics* (Heim 1982, Kamp 1981, Groenendijk & Stokhof 1991).

(4) Giles owns a donkey. He treasures it.

(5) Giles owns a donkey and he treasures it.

- Another kind of anaphora that resists classical treatments.
- More motivation for dynamic semantics.

(6) Either Giles doesn't own a donkey, or he treasures it.

(7) It's not true that Giles doesn't own a donkey! He treasures it.

- Dynamic approach to donkey/discourse anaphora can be extended to account for these cases too (Krahmer & Muskens 1995, Gotham 2019, Mandelkern 2022, Elliott 2020, Hofmann 2022).

Truth conditions of donkey sentences

- First-generation dynamic approaches such as Groenendijk & Stokhof's Dynamic Predicate Logic (DPL) made the following predictions for discourse vs. donkey anaphora.
  - Discourse anaphora gives rise to *existential truth-conditions*, thanks to Egli's theorem: $\exists_x \phi \wedge \psi \iff \exists_x(\phi \wedge \psi)$
  - Donkey anaphora gives rise to *universal truth-conditions*, thanks to Egli's corrolary: $\exists_x \phi \rightarrow \psi \iff \forall_x(\phi \rightarrow \psi)$
  - We'll discuss the truth conditions of bathroom disjunctions later.

(8) Giles owns a donkey and he treasures it.
$\iff$ *Some donkey is owned and treasured by Giles.*

(9) If Giles owns a donkey he treasures it.
$\iff$ *Every donkey owned by Giles is treasured by him.*

In later years, a more nuanced picture emerged: donkey pronouns can be associated with universal *or* existential force (Chierchia 1995, Kanazawa 1994, 2001).

(10) Every farmer who owns a donkey feeds it well.
⇒ *Every farmer who owns a donkey feeds each of his donkeys well.*

(11) No farmer who owns a donkey feeds it well.
⇒ *No farmer who owns a donkey feed any of his donkeys well.*

Although factors such as monotonicity clearly play a role (Kanazawa 1994), context is also important - universal quantifiers are for example sometimes compatible with existential readings.

(12) *Q: Did anyone get wet?*
Everyone who has an umbrella remembered to bring it with them.
$\Longleftrightarrow$ *Everyone who has an umbrella remembered to bring one of their umbrellas*.

This is a heterogeneous reading (Champollion, Bumford & Henderson 2019) , since it is true in a *mixed* scenario, where some may also have an umbrella they didn't bring; previous examples are homogeneous.

Champollion, Bumford & Henderson (2019) develop an account of donkey anaphora, drawing inspiration from work on homogeneity and plural definites (Križ 2017). Their approach can capture both homogeneous and heterogeneous readings, but there are some issues.

- Limited empirical remit: their account is tailored specifically for quantificational environments. I'll suggest that the $\exists/\forall$ distinction is more general than this.
- Their proposal requires an idiosyncratic treatment for sentences with free pronouns.

- I'll argue that the source of ∃/∀-readings lies in violations of the Law of Non-Contradiction (LNC).
- Crucially, if ϕ is *compatible* with $\neg\phi$, then ϕ can be interpreted *exhaustively* (and non-vacuously so) relative to $\neg\phi$.
- The main empirical motivation for this approach will come from ∀/∃-readings in non-quantificational environments, i.e., with discourse anaphora in disjunctive sentences.
- It will turn out that violations of the LNC will be a natural consequence of a principled logic of anaphora which extends to discourse anaphora in disjunctions.

Bathroom disjunctions

Discourse anaphora is typically blocked by disjunction (Groenendijk & Stokhof 1991).

(13) #Either Matthew has a smart shirt, or it's in his closet.

Evans (1977) however noticed that anaphora is possible if *a witness to an indefinite in the initial disjunct is entailed by the first disjunct's negation*.

(14) Either Matthew doesn't have a smart shirt, or it's in his closet.

Just what are the truth-conditions of bathroom disjunctions? Early work suggested that they are *universal*:

(15) Context: *there are two bathrooms in this house; one of them is in a strange place, and one isn't*.
Either there's no bathroom in this house,
or it's in a strange place. (Krahmer & Muskens 1995)

Expected to be false based on universal truth-conditions:

$$\forall_x(\textbf{Bathroom}(x) \to \textbf{Strange}(x))$$

In the right context however, bathroom disjunctions can have an existential reading (Elliott 2023b).

(16) Context: *We're wondering whether Gabe paid with cash or card.*
Either Gabe doesn't have a credit card with him,
or he paid with it.

- *False* if Gabe has credit cards with him, and didn't pay with *any of them*.
- *True* if Gabe has credit cards and paid with at least one of them.
- The proposed truth-conditions:

$$\neg\exists_x \mathbf{Card}(x) \vee \exists_x(\mathbf{Card}(x) \wedge \mathbf{Pay}(x))$$

Existing accounts: an impasse

- Krahmer & Muskens (1995) develop a variant of DRT tailored to derive only *universal* readings.
- Elliott (2022) develops a trivalent dynamic semantics which derives only *existential* readings.
    - Mandelkern (2022), Hofmann (2019, 2022) don't explicitly discuss the truth-conditions of bathroom disjunctions in detail, but seem to also derive existential truth conditions(?).
- There are also accounts which claim that bathroom disjunctions entail *uniqueness* (Gotham 2019), but sage-plant sentences show putative uniqueness inferences are cancellable (Mandelkern & Rothschild 2020).

In essence, (Krahmer & Muskens 1995) and (Elliott 2022) differ in which of the following (classically equivalent!) formulas captures bathroom disjunctions.

(17) $\neg\phi \vee (\phi \rightarrow \psi)$

(18) $\neg\phi \vee (\phi \wedge \psi)$

- Egli's theorem/corrolary mean that they give rise to different truth-conditions, if an existential in ϕ binds a pronoun in ψ.
- In the following, I show that context can militate between ∀- and ∃-readings.

(19) Context: *spoken by a donkey welfare activist organizing boycotts against farmers who aren't kind to their donkeys:*
Every farmer who doesn't have a donkey or feeds it treats is safe from reprisal.

The *donkey welfare activist* is insinuating that any farmer who neglects any of his donkeys is subject to reprisal.

$$\forall_f(\overbrace{(\neg\exists_d \mathbf{Own}(f,d) \vee \forall_d(\mathbf{Own}(f,d) \rightarrow \mathbf{Treats}(f,d)))}^{\text{Partee disjunction}} \rightarrow \mathbf{Safe}(f))$$

(20) Context: *spoken by a cruel industrialist who believes that starving donkeys are the most efficient workers.*
Every farmer who doesn't have a donkey or feeds it treats is working below maximum efficiency.

The *cruel industrialist* is insinuating that a farmer who indulges *any* of his donkeys is working below maximum efficiency.

$$\forall_f(\overbrace{(\neg\exists_d \mathbf{Own}(f,d) \lor \exists_d(\mathbf{Own}(f,d) \land \mathbf{Indulge}(f,d))}^{\text{Partee disjunction}} \rightarrow \mathbf{BelowMax}(f))$$

- In the following, I'll develop a logic - *existential dynamic semantics* - which derives heterogeneous readings *everywhere*.
- In this logic, violations of the LNC are possible.
- Homogeneous readings will be derived by strengthening ϕ relative to $\neg\phi$.

Existential Dynamic Semantics

- In previous work, I developed an account of anaphoric accessibility based on a *predictive* theory of presupposition projection by embedded Strong Kleene trivalent semantics in a dynamic setting Elliott (2020, 2022, 2023a) .
  - There are two variations: Existential Dynamic Semantics (Elliott 2023a) and Bilteral Update Semantics (Elliott 2022); I use the former here because it's closest to classical DPL.

- The semantics is bilateral - An expression ϕ is associated with:
  - A positive update $[\phi]^+$.
  - A negative update $[\phi]^-$
- Updates are functions from *world-assignment pairs* to *sets of assignments* (Groenendijk & Stokhof 1991, Groenendijk, Stokhof & Veltman 1996).
- The positive update $[\phi]^+$ often (but not always) corresponds to the interpretation of ϕ in DPL.

(21) $[\text{it}_x \text{ is upstairs}]^+ := \lambda(g, w) \,.\, \{ h \mid h = g, g_x \text{ is upstairs in } w \}$

(22) $[\text{it}_x \text{ is upstairs}]^- := \lambda(g, w) \,.\, \{ h \mid h = g, g_x \text{ isn't upstairs in } w \}$

- Atomic sentences are associated with positive/negative updates which test whether possibilities are ones at which the sentence is true/false respectively.
  - I assume that assignments are partial, which means that there may be some (w, g), s.t., $[\phi]^+(w, g), [\phi]^-(w, g) = \varnothing$.

In a dynamic setting, the context set includes not just information about the world, but also anaphoric *information*.

Concretely, I take the context set to be a set of world-assignment pairs (Heim 1982).

This leads to a natural generalization of "Stalnaker's bridge" (von Fintel 2004), encompassing anaphoric presuppositions:

(23) Assertion

a. A sentence ϕ is assertable at a context set c iff
$\neg\exists(w,g) \in c[[\phi]^{+}(w,g), [\phi]^{-}(w,g) = \varnothing]$

b. If assertable, the result of update c with ϕ is
$c + \phi := \bigcup_{(w,g)\in c} [\phi]^{+}(w,g)$

Let's see how this derives Heimian familiarity.

(24) $[P(x)]^{+}(w_b, [x \to b]) = \{ (w_b, [x \to b]) \}$

(25) $[P(x)]^{-}(w_b, [x \to b]) = \varnothing$

(26) $[P(x)]^{+}(w_b, [y \to b]) = \varnothing$

(27) $[P(x)]^{-}(w_b, [y \to b]) = \varnothing$

A sentence with a free variable x is only assertable if *every* possibility in the context set provides a value for x.

A sentence with a free variable x presupposes that x has been introduced as a discourse referent.

The positive update of an existential statement introduces a discourse referent, just like in ordinary dynamic semantics.

(28) [there is a^{x} bathroom]$^{+}$ =
$\lambda(w, g)\,.\,\{\, h \mid g[x]h, h_x \text{ a bathroom in } w \,\}$

Importantly, the negative update of an existential statement is a test, i.e., it doesn't introduce any anaphoric information (see Mandelkern 2022 for a closely related idea).

(29) [there is a^{x} a bathroom]$^{-}$ =
$\lambda(w, g)\,.\,\{\, h \mid h = g, g_x \text{ isn't a bathroom in } w \,\}$

Double Negation Elimination (DNE) is validated by a standard "flip-flop" entry for negation:

(30) $[\text{not}\,\phi]^{+} = \lambda(w, g)\,.\,[\phi]^{-}(w, g)$

(31) $[\text{not}\,\phi]^{-} = \lambda(w, g)\,.\,[\phi]^{+}(w, g)$

It's hopefully immediately obvious that this entry validates DNE.

In first-generation dynamic semantics, conjunction is responsible for passing anaphoric information from left-to-right. This was accomplished by treating conjunction as a kind of *relational composition* (Groenendijk & Stokhof 1991).

Since I'm taking the world of evaluation into account, I'll use this slightly tweaked definition of relational composition to compose updates.

(32) Update composition:

$$m \,;\, n \;:=\; \lambda(w, g)\,.\, \bigcup \{\, m(w, h) \mid h \in n(w, g) \,\}$$

$m \,;\, n$ constructs a new update, by feedings the outputs of m pointwise into n, and gathering up the results.

The main insight of Externally-dynamic Dynamic Semantics (EDS) is that relational composition can be used as part of a *recipe for constructing updates* associated with complex sentences, based on the Strong Kleene logic of indeterminacy.

Strong Kleene logic is a trivalent logic that emerges when the third truth value is interpreted as *maybe true and maybe false*.

In order for a conjunctive sentence $\phi \wedge \psi$ to be true, both conjuncts must be true.

In EDS, this means that we construct the positive update by composing the positive updates of each conjunct; no other updates are relevant.

(33) $[\phi \wedge \psi]^+ := [\phi]^+ ; [\psi]^+$

This delivers identical results to conjunction in first-generation dynamic theories; a discourse referent introduced by ϕ can satisfy the anaphoric presupposition of ψ, but not vice versa.

I'll defer the negative update of conjunction, and move straight on to the positive update of disjunction, which will involve allowing for uncertainty.

In order to model disjunction, we'll need an update to stand in as a proxy for the third truth-value in trivalent logic.

We'll do this by defining presupposition failure a test for undefinedness:

(34) $[\phi]^{\#} := \lambda(w, g) \,.\, \{\, h \mid h = g, [\phi]^{+}(w, g), [\phi]^{-}(w, g) = \varnothing \,\}$

The test succeeds (i.e., has a non-empty output), if both the positive and negative outputs are empty.

In BUS, we cash out the Strong Kleene truth table as a recipe for constructing positive/negative updates of complex expressions.

$\phi \vee \psi$	ψ_+	ψ_-	$\psi_\#$
ϕ_+	$+$	$+$	$+$
ϕ_-	$+$	$-$	$\#$
$\phi_\#$	$+$	$\#$	$\#$

Figure 1: Strong Kleene disjunction

We'll take the union of the + cells for the positive update, and the − cells for the negative update. For the positive update, this necessarily involves composing with the failure test.

To get the positive update of disjunction, we collect the updates corresponding to true cells in the Strong Kleene truth-table.

(35) $$[\phi \vee \psi]^+ := \overbrace{[\phi]^+ ; [\psi]^+ \cup [\phi]^+ ; [\psi]^- \cup [\phi]^+ ; [\psi]^\#}^{\text{ver. via } \phi} \underbrace{\cup [\phi]^- ; [\psi]^+ \cup [\phi]^\# ; [\psi]^+}_{\text{ver. via } \psi}$$

In the next section, I'll show how this update correctly captures (anaphoric) presupposition satisfaction in bathroom disjunctions, and ∃-readings.

N.b. union here is understood as *relational*, i.e., given two updates p and q, $p \cup q := \lambda g \,.\, p(g) \cup q(g)$

(36) Either there's no bathroom in this house, or it's in a strange place.
$\neg\exists_x B(x) \vee S(x)$

We now have a recipe for computing the positive update associated with this sentence.

(37) $[\neg\exists_x B(x) \vee S(x)]^+ :=$
$$[\neg\exists_x B(x)]^+ ; [S(x)]^+ \cup [\neg\exists_x B(x)]^+ ; [S(x)]^- \cup [\neg\exists_x B(x)]^+ ; [S(x)]^\# \cup [\neg\exists_x B(x)]^- ; [S(x)]^+ \cup [\neg\exists_x B(x)]^\# ; [S(x)]^+$$

First, we can simplify based on flip-flop negation; Let's start with the first line:

(38) $[\neg\exists_x B(x) \vee S(x)]^+ :=$
$[\exists_x B(x)]^- ; [S(x)]^+ \cup [\exists_x B(x)]^- ; [S(x)]^- \cup [\exists_x B(x)]^- ; [S(x)]^\#$
$\cup [\exists_x B(x)]^+ ; [S(x)]^+ \cup [\exists_x B(x)]^\# ; [S(x)]^+$

- Recall that the negative update of an existential statement essentially tests whether there is no witness.
- One of $[S(x)]^{+,-,\#}$ will invariably be non-empty depending on whether x is defined at the input assignment; $S(x)$ is a test so it introduces no new information.
- $\therefore$ the first line is equal to $[\exists_x BH(x)]^-$

(39) $[\neg\exists_x B(x) \vee S(x)]^+ :=$
$[\exists_x B(x)]^- ; [S(x)]^+ \cup [\exists_x B(x)]^- ; [S(x)]^- \cup [\exists_x B(x)]^- ; [S(x)]^\#$
$\cup [\exists_x B(x)]^+ ; [S(x)]^+ \cup [\exists_x B(x)]^\# ; [S(x)]^+$

- $[\exists_x B(x)]^\#$ will be empty for any input, since there are no free variables (i.e., no possibility of presupposition failure).
- $\therefore$ the second line is equal to $[\exists_x B(x)]^+ ; [S(x)]^+$.
    - This is a simple case of (conjunctive) discourse anaphora.

- The positive update of the sentence corresponds to the union of the first line, and the second line, so, with simplifications:

$$
\begin{aligned}
&[\neg\exists_x B(x) \vee S(x)]^+ \\
(40)\quad &= [\exists_x B(x)]^- \cup [\exists_x B(x)]^+ \,;\, [S(x)]^+ \\
&= \lambda(w, g)\,.\,\{\, h \mid h = g, \mathbf{B}_w = \varnothing \,\} \cup \{\, h \mid g[x]h, h_x \in \mathbf{B}_w \cap \mathbf{S}_w \,\}
\end{aligned}
$$

- Given a possibility (w, g) the bathroom disjunction has a non-empty output iff either (a) there's no bathrooms in w, or (b) there's a bathroom in a strange place in w.
- Note especially that the update in (40) conditionally introduces a *bathroom-strange-place* discourse referent.

- The predicted truth-conditions clearly correspond to the ∃-reading, assuming a standard dynamic notion of truth.

(41) ϕ is true at (w, g) if $[\phi]^{+}(w, g) \neq \varnothing$

- One might expect that the truth-conditions of a negated bathroom disjunction are therefore strong, but not so; negated bathroom disjunctions correspond to a simple case of conjunctive discourse anaphora.

(42)
$$\begin{aligned}[\neg\exists_x B(x) \vee S(x)]^{-} &= [\neg\exists_x B(x)]^{-} ; [S(x)]^{-} = [\exists_x B(x)]^{+} ; [S(x)]^{-} \\ &= \lambda(w, g) \,.\, \{\, h \mid g[x]h, h_x \in \mathbf{B}_w, \notin \mathbf{S}_w \,\}\end{aligned}$$

Weak truth-conditions for negated bathroom disjunctions is a reflex of validating de Morgan's laws and DNE!

(43) Neither is there no bathroom, nor is it upstairs.
⇒ There's not no bathroom, and it's not upstairs.
⇒ There's a bathroom, and it's not uptairs.

Although logically neat, is this a good prediction?

(44) Context: *There's two bathrooms in this house; one upstairs, and one downstairs.*

a. #Neither is there no bathroom in this house, nor is it upstairs.
b. There's a bathroom in this house, and it's not upstairs.

Nevertheless, massaging the context a little, it seems to be possible:

(45) A: *I think Sarah is going to get wet*
— she either doesn't own an umbrella, or she left it at home.
B: No — neither does Sarah not own an umbrella,
nor did she leave it at home.
It's actually in her rucksack.

- A plausible intuition: B's assertion isn't falsified, if Sarah has multiple umbrellas, and brought one but left the others at home.
  - In perhaps the limiting case, we'd like our semantics to allow for heterogeneous readings of negated bathroom disjunctions.

- A standard Strong Kleene trivalent semantics for material implication validates $\phi \to \psi \iff \neg\phi \vee \psi$.
- Constructing an update for the implication based on the Strong Kleene truth table then, results in existential truth-conditions.

(46) $[\exists_x B(x) \to U(x)]^+ = [\neg\exists_x B(x) \vee U(x)]^+$

- True if either (a) there's no bathroom, or (b) there's a bathroom upstairs.
- As we've discussed, being able to generate the ∃-reading for donkey sentences is desirable.

- EDS validates de Morgan's laws, so the predictions for negated conjunctions are again, existential readings:

(47) $[\neg(\exists_x B(x) \land U(x))]^+ = [\neg\exists_x B(x) \lor \neg U(x)]$

- True if either (a) there's no bathroom, or (b) there's a bathroom which isn't upstairs.
- Seems doubtful as the default reading for negated conjunctions: "I doubt that there's a bathroom and it's upstairs." (*I believe that if there's any bathrooms, none of them are upstairs*)

- A projection algorithm based on Strong Kleene trivalent semantics predicts *heterogeneous readings* everywhere:
- $\neg\exists_x B(x) \lor U(x)$:
  - *if there's a bathroom, then there's a bathroom upstairs*
- $\neg(\neg\exists_x B(x) \lor U(x))$:
  - *there's a bathroom which isn't upstairs.*
- $\exists_x B(x) \land U(x)$:
  - *there's a bathroom which is upstairs.*
- $\neg(\exists_x B(x) \land U(x))$:
  - *if there's a bathroom then there's a bathroom which isn't upstairs.*
- Next step: deriving homogeneous readings in a context-sensitive fashion.

- First, we'll show how to derive homogeneous readings, via a pragmatic principle.
  - Our initial attempt will however *only* derive homogeneous readings.
- Subsequently, we'll show how to account for the availability of hetereogeneous readings given the right context, by relativizing the pragmatic principle to a QuD (Roberts 2012, Champollion, Bumford & Henderson 2019).

Homogeneous readings

Look again at the predicted truth conditions for bathroom disjunctions and their negations; they are compatible!

- $\neg\exists_x B(x) \vee U(x)$:
    - *if there's a bathroom, then there's a bathroom upstairs*
- $\neg(\neg\exists_x B(x) \vee U(x))$:
    - *there's a bathroom which isn't upstairs.*
- This shows that in EDS, the LNC doesn't hold.
- $\neg\exists_x B(x) \vee U(x) \wedge \neg(\neg\exists_x B(x) \vee U(x))$
    - *there's a bathroom which is upstairs, and a bathroom which isn't upstairs.*

This initially seems like a strange result, but we'll use it to account for homogeneous readings by appealing to a pragmatic principle.

Polar questions can be understood induce an equivalence relation between worlds (Groenendijk & Stokhof 1984).

(48) Is it raining?

(49) $\lambda w\,.\,\lambda w'\,.$ it's raining in $w \iff$ it's raining in w'

- Cell 1: *raining possibilities.*
- Cell 2: *Not-raining possibilities.*

In the analysis of *wh*-questions, it's common to *derive* this equivalence relation from a set of alternatives corresponding to the Hamblin denotation of the question (Heim 1994).

(50) $w \sim w' \iff \forall p \in Q[p(w) = p(w')]$

In the case of polar questions, we can think of the alternatives in Q as corresponding to the *yes*, and *no* answers to the question.

(51) { it's raining, it's not raining }

This of course gives rise to the same two cells as before; however, in a logic where ϕ and $\neg\psi$ may be compatible, (50) will do some interesting work.

Now consider how we might derive an equivalence relation from the following alternatives:

$$\left\{ \begin{array}{l} ALT^+ : \text{either there's no bathroom or it's upstairs,} \\ ALT^- : \text{neither is there no bathroom, nor is it upstairs} \end{array} \right\}$$

Equivalently:

$$\left\{ \begin{array}{l} ALT^+ : \exists_x B(x) \rightarrow \exists_x (B(x) \wedge U(x)), \\ ALT^- : \exists_x (B(x) \wedge \neg U(x)) \end{array} \right\}$$

Polar partitioning principle cont.

This gives rise to *three* cells(!); *yes* and *no* answers identify cells 1 and 2 respectively.

- Cell 1 (ALT^+ true, ALT^- false): contains possibilities where every bathroom (if any) is upstairs.
  - $\exists_x B(x) \rightarrow \exists_x (B(x) \wedge U(x)) \wedge \neg\exists_x (B(x) \wedge \neg U(x))$
  - $\Rightarrow \forall_x (B(x) \rightarrow U(x))$
- Cell 2 (ALT^+ false, ALT^- true): contains possibilities where every bathroom (if any) is downstairs.
  - $\exists_x (B(x) \wedge \neg U(x)) \wedge \neg(\exists_x B(x) \rightarrow \exists_x (B(x) \wedge U(x)))$
  - $\Rightarrow \forall_x (B(x) \rightarrow \neg U(x))$
- Cell 3 (both true): contains possibilities where at least one bathroom is upstairs, and at least one bathroom is downstairs.
  - $\exists_x (B(x) \rightarrow \exists_x (B(x) \wedge U(x)) \wedge \exists_x (B(x) \wedge \neg U(x))$
  - $\Rightarrow \exists_x, y(B(x) \wedge B(y) \wedge U(x) \wedge \neg U(y))$

- The intuition behind our pragmatic principle is that a sentence ϕ, when asserted, is (by default) interpreted *exhaustively* relative to the polar question induced by $\{\phi, \neg\phi\}$.
- In a classical setting, this is trivial - due to the LNC, if ϕ is true, then $\neg\phi$ is false, so ϕ uniquely identifies the ϕ cell.
- As we've seen, the LNC doesn't hold in EDS: ϕ and $\neg\phi$ can both be true, then interpreting ϕ exhaustively amounts to using ϕ to identify the cell in which ϕ is true and $\neg\phi$ isn't.

- For expository purposes, we can build exhaustive interpretation directly into our bridge principle as follows; sentences are assumed to be "just true".

(52) Exhaustive assertion (v1):
A sentence ϕ is *assertable* at a Heimian context set c iff $\forall (w, g) \in c[[\phi]^{\#} = \varnothing]$. If assertable, the result of *asserting* a sentence ϕ at c,
$c + \phi := \bigcup \{ [\phi]^{+}(w, g) \mid [\phi]^{-}(w, g) = \varnothing, (w, g) \in c \}$

Ultimately, this should be folded into an independently-needed account of exhaustive interpretations.

This initial attempt derives homogeneous readings *everywhere*.

(53) $$c + [\neg\exists_x B(x) \vee U(x)] = \bigcup \left\{ [\neg\exists_x B(x) \vee U(x)]^+(w, g) \;\middle|\; \begin{array}{l} [\exists_x B(x) \vee \neg U(x)]^+(w, g) = \varnothing \\ , (w, g) \in c \end{array} \right\}$$

Updating *c* with “either there’s no bathroom or it’s upstairs” throws out possibilities where there is a bathroom which isn’t upstairs, and retains possibilities where there isn’t a bathroom, or all bathrooms are upstairs, conditionally introducing a discourse referent.

The same result holds for the conditional “If there’s a bathroom then it’s upstairs”, due to $\neg\phi \vee \psi \iff \phi \rightarrow \psi$

As we’ve seen, we *don’t* want to have homogeneous readings everywhere, so we need a way of weakening the predictions of exhaustive interpretation, relative to contextual factors.

Here, we use the same technique as (Križ 2017, Champollion, Bumford & Henderson 2019) and model contextual relevance as a salient *Question under Discussion* (QuD) (Roberts 2012).

We assume that assertion is always made relative to a (possibly implicit) question, which is modelled as a contextually salient equivalence relation $\sim_Q$.

(54) A sentence ϕ is *assertable* at a Heimian context set c iff $\forall (w, g) \in c[[\phi]^{\#} = \varnothing]$. If assertable, the result of *asserting* a sentence ϕ at c relative to a question Q:
$c + \phi := \bigcup \{ [\phi]^{+}(w, g) \mid \exists w' \sim_Q w, [\phi]^{-}(w', g) = \varnothing, (w, g) \in c \}$

- Relative to a maximally specific *fact-finding* questions, where no two worlds are Q-equivalent, the QuD makes no difference and homogeneous readings are derived.
- Heterogeneous readings are derived by taking a broader QuD, as we'll show in more detail in the following.

- Imagine a context where the QuD is: *how did Gennaro pay for dinner*.
    - The context is divided into cells according to how Gennaro paid (i) a cell containing worlds in which Gennaro paid with a credit card, and in no other way, (ii) a cell containing worlds in which Gennaro paid with cash, and in no other way.
    - Note that the question doesn't draw any distinctions according to how many credit cards Gennaro has.

Now consider an assertion of the following sentence, in light of its negative contribution:

(55) Either Gennaro doesn't have a credit card, or he paid with it.

(56) Neither does Gennaro have a credit card, nor did he pay with it.
⇒ *Gennaro has a credit card that he didn't pay with*

- Assertion will leave possibilities in which Gennaro has multiple credit cards and just paid with one of them, so long as it's contextually possible that there is no credit card which wasn't used for paying.
    - Since the modified bridge makes these possibilities Q-equivalent, the hereogeneous reading is derived.

(57) Context: *Did Josie get wet? I bet she either doesn't have an umbrella, or left it at home.*
Neither does Josie have no umbrella, nor did she leave it at home!

- If the QuD distinguishes worlds in which Josie got wet, from worlds in which she didn't.
  - The distinction between worlds in which Josie brought one her umbrellas, and left the others at home, and worlds in which she brought all her umbrellas is elided.
  - What's relevant is whether she left *all of her umbrellas* at home.
  - This is a *heteogeneous ∀-reading*.

Conclusion

- One striking (and on the face of it, wrong) prediction is that *discourse anaphora* should give rise to homogeneous readings, just in case we can retrieve a suitable QuD (but see (Chatain 2018)).
- Another clear question is whether there are other environments which lead to compatibility of ϕ and $\neg\phi$ in natural language.
    - One intriguing possibility is to analyze sentences with definite plurals and their negations as as such a case.
    - Interpreting each of the following sentences exhaustively relative to its polar opposite derives homogeneity straightforwardly.

(58) The boys left. *at least one boy left.*

(59) The boys didn't leave. *at least one boy didn't leave.*

- I've shown that the famous ∃/∀ ambiguity observed in donkey sentences is a more general phenomenon; it can be detected in other cases of cross-sentential anaphora.
- This suggests that a general account is called for. We showed that any logical theory of accessibility with the following properties gives rise to violations of the LNC:
  - Derives existential truth conditions for bathroom disjunctions.
  - Validates de Morgan's equivalences.
- A theory which gives rise to violations of the LNC automatically gives rise to new possibilities wrt exhaustive interpretation.

Fin

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