# Identifying discourse referents

Dynamic semantics meets inquisitive semantics

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1 Resources

- (Dotlačil & Roelofsen 2019)
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- (Dotlačil & Roelofsen 2021b)

2 Initial motivations

2.1 Background

In this class, we’ve explored theories of anaphora which rely on a fine-grained notion of information, which encodes contextual (un)certainty about the values of variables (i.e., discourse referents). Asserting an existential statement:

1. Narrows down the worldly possibilities.
2. Expands the range of anaphoric possibilities.

In a Heimian world, asserting an existential statement both (i) narrows down the worldly possibilities, and (ii) expands the range of anaphoric possibilities.

Most work in dynamic semantics doesn’t look beyond a classical Stalnakerian model of discourse, where Heimian file contexts are used to model the context set.

Heimian file contexts, while extending the classical notion of an information state, aren’t expressive enough to capture issues raised by interrogative sentences.

2.2 Initial motivations: questions and anaphora

In truth-conditional semantics, questions are typically analyzed in terms of their resolution conditions (Hamblin 1973, Karttunen 1977, Groenendijk & Stokhof 1984).

Initial motivations for a dynamic approach to questions (Groenendijk 1998, Haida 2007): there is more to the meaning of a question than just its (classical) resolution conditions, as shown by interrogative variations of Partee’s famous marble sentences:

(1) Of my six marbles, which\(^x\) one is not in the bag? Is it\(^x\) under the sofa?
(2) Of my six marbles, which five are in the bag? #Is it\(^x\) under the sofa?
2.3 Complex interrogatives and accessibility

*Wh*-expressions, just like indefinites, can introduce discourse referents. A subsequent pronoun can (but need not) be itself embedded in a question.

(3) Who\(\hat{x}\) is at the door? How long have they\(x\) been waiting for?
(4) Who\(\hat{x}\) is at the door? They\(x\)’re knocking extremely loudly.

Additionally, indefinites in polar questions can license discourse anaphora (in conjunctive questions).

(5) Did you read a paper, and did you enjoy it?

The conditions under which indefinites in questions license discourse anaphora in a subsequent declarative relate to the generalization we’ve explored for much of this semester - a pronoun is licensed just in case a discourse referent is contextually entailed (see also (Mandelkern 2022)).

(6) A: Does Andreea have a\(x\) husband? B: Yes, she’s married. He\(x\)’s waiting outside.
(7) A: Does Andreea have a\(x\) husband? B: ??No, she isn’t married. He\(x\)’s waiting outside.

Indefinites inside of *wh*-interrogatives can license discourse anaphora too:

(8) Who\(\hat{x}\) is presenting a\(y\) paper, and are they\(x\) presenting it\(y\) on Wednesday?

One manifestation of this is the availability of so-called “e-type” readings of quantifiers under ellipsis (Merchant 2001), explored in detail from a dynamic perspective by (Elliott & Sudo 2016).

(9) If John asks me *how* a\(x\) mathematical theorem was proved, I will also tell him by whom…
   a. ??...a\(x\) mathematical theorem was proved.
   b. ...the\(x\) mathematical theorem was proved.

This observation extends to discourse plurals:

(10) John applied to five\(x\) graduate schools, but I don’t know why…
   a. ...John applied to five graduate schools.
   b. ...John applied to the\(x\) five graduate schools.

\(^1\)The analysis (roughly) is that ellipsis of a clause is possible if it stands in a mutual dynamic entailment relation (modulo focus) with its antecedent.
In addition to discourse anaphora, *donkey* anaphora is available in *conditional questions*; unsurprisingly, both weak and strong readings are available.

(11) If a\textsuperscript{x} farmer owns a\textsuperscript{y} donkey, will he\textsubscript{x} beat it\textsuperscript{x}?
(12) If you have a\textsuperscript{x} credit card, would you mind paying with it\textsubscript{x}?

More generally, it seems that accessibility in complex questions patterns with accessibility in complex declaratives (see especially [Enguehard 2021] on parallel presupposition projection facts).

(13) Which\textsuperscript{x} paper did you read and did you enjoy it\textsubscript{x}?
(14) Did you read a\textsuperscript{x} paper, and did you enjoy it\textsubscript{x}?
(15) Did you NOT read a\textsuperscript{x} paper, or did you enjoy it\textsubscript{x}?
(16) If you read a\textsuperscript{x} paper, did you enjoy it\textsubscript{x}?

### 2.4 Intervention effects

Examples from [Beck 1996, 2006].

(17) Wer hat Luise wo angetroffen?
    Who has Luise where met?
(18) *Wer hat niemanden wo angetroffen?
    Who has nobody where met?
(19) Wer hat wo niemanden angetroffen?
    Who has where nobody met?

Suspicion of the dynamic approach to intervention effects [Haida 2007, Honcoop 1998]:

- The operators that are responsible for intervention effects are just those that are *externally static* in the sense of [Groenendijk & Stokhof 1991b].

### 2.5 The plan

- *Inquisitive semantics* provides a framework in which a uniform notion of information captures both what is mutually known, and what is mutually taken to be at issue. This makes it extremely well-suited to providing a uniform account of how assertions/questions contribute information to the context set, as explored in a wide range of works [Ciardelli, Groenendijk & Roelofsen 2019].
- Combine the basic ideas of dynamic semantics and inquisitive semantics (based on [Dotlačil & Roelofsen 2019]).
3 Inquisitive semantics: the basics

One of the basic ideas behind inquisitive semantics is that contexts have additional structure - instead of treating a context as an information state, we treat contexts as downward closed sets of information states\(^2\).

Such structures are expressive enough to capture issues while subsuming (classical) informational content.

- Concretely, the informational content of an inquisitive context \(C\) is just the union of all \(s \in C\).
- What’s at issue is represented by the alternatives, i.e., the maximal elements of \(C\); a context is inquisitive just in case it includes more than one alternative.

In a standard inquisitive setting, the initial context is simply the powerset of the ignorance state. We can handily represent inquisitive contexts using diagrams where maximal states are highlighted.

The middle diagram illustrates an inquisitive context with two alternatives - the result of updating the initial context with “a or b”.

Unlike partition semantics, inquisitive semantics by default gives rise to weak resolution conditions - the issue raised by “a or b” is resolved by the proposition “that a” and the proposition “that b” (no exclusivity inference).

Atomic sentences denote downward closed sets of states, i.e., the states which support the sentence. Update is cashed out simply as set intersection (just as in a Stalnakerian setting).

\[
[a] := \{ s | s \vdash a \}
\]

Disjunctive sentences denote the union of the states that support each disjunct; when we update a context with a disjunctive sentence, we get back an inquisitive context where each disjunct is reflected in an alternative.

\[
[\phi \lor \psi] := [\phi] \cup [\psi]
\]

\(^2\)A set \(C\) is downward closed iff for each element \(s \in C\), subsets of \(s\) are necessarily elements of \(C\) too (including the empty set!).
Conjunction is just intersection:

\[
\begin{align*}
\begin{array}{c}
w_a \quad w_a \\
\hline
w_b \quad w_\emptyset
\end{array}
\end{align*}
\quad \xrightarrow{\lor b} 
\begin{align*}
\begin{array}{c}
w_a \quad w_a \\
\hline
w_b \quad w_\emptyset
\end{array}
\end{align*}
\]

(22) \( [\phi \lor \psi] := [\phi] \cap [\psi] \)

Negation closes off issues (defined derivatively here in terms of info):

(23) \( [\text{info}(\phi)] := P(\bigcup [\phi]) \)

(24) \( [\neg \phi] := P(W - [\text{info}(\phi)]) \)

4 Inquisitive dynamic semantics (Dotlačil & Roelofsen 2019)

4.1 The basic system

In standard inquisitive semantics, information states are sets of possible worlds; in dynamic inquisitive semantics, we replace this notion with Heimian states.

We’ll start by giving a static notion of support for atomic sentences.

**Definition 4.1. Support.** Support for atomic sentences.

\[
s \vdash P(x_1, \ldots, x_n) \iff \forall (w, g) \in s, (g(x_1), \ldots, g(x_n)) \in I_w(P)
\]

**Definition 4.2. Atomic sentences.** If \( \phi \) is atomic, then:

- \( C[\phi] = \{ s \in C \mid s \vdash \phi \} \)

As before, we’ll continue to assume that assignments are partial.

- We need to say what happens if an atomic sentence isn’t defined at a possibility in some information state.
- Following (Dotlačil & Roelofsen 2019), we’ll assume that if a sentence \( \phi \) is undefined at any possibility in any state \( s \in C \), the result is undefinedness (implicitly, a weak Kleene logic).

**Definition 4.3. Conjunction.** As in update semantics, conjunction is interpreted as a successive update.
Discourse referents are introduced state-by-state via random assignment.

**Definition 4.4. Random assignment** (contexts). Random assignment at a context performs random assignment pointwise on states\(^3\)

- \( C[\varepsilon_x] := \{ s[\varepsilon_x] \mid s \in C \} \)

In effect, an atomic sentence with a variable \( x \) presupposes at \( C \) familiarity of \( x \) at every state in \( C \); random assignment guarantees familiarity at every state.

The diagram below illustrates how random assignment (a) expands states and (b) an atomic sentence cuts the possibilities back down. N.b. none of the resulting contexts are inquisitive (yet).

\[ \begin{array}{c|cccc|c}
| & w_a & w_{ab} & w_b & w_b & \varepsilon_x \\
\hline
[ & \bullet & \bullet & \bullet & \bullet & \rightarrow [x \rightarrow a] \\
[ & \bullet & \bullet & \bullet & \bullet & \rightarrow [x \rightarrow b] \\
\end{array} \]

**Definition 4.5. Disjunction.** Disjunction parallels inquisitive disjunction and DPL’s program disjunction, and as such gives rise to an inquisitive context.

- \( C[\phi \lor \psi] = C[\phi] \cup C[\psi] \)

It’s important to note that the disjunction that (Dotlačil & Roelofsen 2019) assume is internally static but externally dynamic.

### 4.2 Issues about discourse referents

Having information about the values of variables encoded in states allows us to define an interesting operation – \(?_x\), which raises an issue about the value of \( x \) by inducing alternatives that agree on \( x \).\(^4\)

\(^3\)We assume the standard definition of random assignment for states:

\( s[\varepsilon_x] := \{ (w, h) \mid g[x] h \land (w, g) \in s \} \)

\(^4\)Here we make use of an equivalence relation over possibilities \( \sim_x \), which is defined as follows:

\( (w, g) \sim_x (w', h) \iff g_x = h_x \)

Crucially, it’s not necessary that \( w = w' \).
Definition 4.6. Witness identification operator.

- \( C[?x] := \{ s \in C \mid \forall i, i' \in s, i \sim_x i' \} \)

\[
\begin{align*}
[w_a \quad w_{ab} \quad w_b \quad w_0] & \quad \overset{\varepsilon_x \wedge P(x)}{\Rightarrow} \quad [x \rightarrow a] \quad \overset{?x}{\Rightarrow} \quad [x \rightarrow a] \\
[x \rightarrow b] & \quad \Rightarrow \quad [x \rightarrow b] \quad \overset{?x}{\Rightarrow} \quad [x \rightarrow b]
\end{align*}
\] (1)

Inquisitive existential quantification can be defined syncategorematically in terms of random assignment and variable identification. \( \exists_x \phi \) does the following:

- Randomly assigns values to \( x \), as restricted by \( \phi \).
- Raises an issue about the value of \( x \).

\[
(25) \quad \exists_x \phi := (\varepsilon_x \wedge \phi) \wedge ?x
\]

4.3 Discharging issues

In standard inquisitive semantics non-inquisitive closure (!) closes off issues associated with an inquisitive context. Its definition is simple:

Definition 4.7. Non-inquisitive closure (contexts).

- \( !C = P(\text{info}(C)) \)

In inquisitive dynamic semantics, we can define a closure operator ! that closes off issues in its scope but not discourse referents.


- \( C[!\phi] := \{ s' \in C[\phi] \mid \exists s \in C, s \leq s' \} \)

This allows us to close off the issues raised by an existential statement while retaining the introduced discourse referents. Moreover, the following are equivalent:
\[(26) \quad \neg(\varepsilon_x \land P(x) \land ?_x)\]
\[(27) \quad \varepsilon_x \land P(x)\]

This provides tools for distinguishing between the semantic contribution of an indefinite vs. an interrogative.

Negation on the other hand is defined in such a way that in closes off issues (as negation does in inquisitive semantics) and discourse referents (as negation does in dynamic semantics):

**Definition 4.9. Negation.** Negation of \(\phi\) at \(C\) returns the states in \(C\) which don’t have a consistent substate that subsists in \(C[\phi]\)

- \(C[\neg\phi] := \{ s \in C \mid \neg \exists t \neq \emptyset \land t \subseteq s \land t \prec C[\phi] \}\)

Just as in standard inquisitive semantics, issues can be raised by disjunction. Inquisitive closure is used to analyze polar questions.

**Definition 4.10. Inquisitive closure.**

- \(?\phi := \phi \lor \neg\phi\)

### 4.4 Implication

**Definition 4.11. Implication.**

- \(C[\phi \rightarrow \psi] := \{ s \in C \mid \forall t \subseteq s, each \ descendant \ of \ t \ in \ C[\phi] \ subsists \ in \ C[\phi][\psi] \}\)

### 4.5 Empirical payoff

#### 4.5.1 Anaphora between questions

\[(28) \quad \text{Which}^z \text{ man read a}_y \text{ book? Did he}_x \text{ like it}_y?\]
\[(29) \quad \varepsilon_x \land \varepsilon_y \land M(x) \land B(y) \land R(x, y) \land ?_x \land \underbrace{?(L(x, y))}_{\text{wh-Q}} \land \underbrace{?((L(x, y)))}_{\text{pol-Q}}\]

- The first conjunct introduces a man dref \(x\) and a book dref \(y\), s.t., \(x\) read \(y\), and raises an issue about the value of \(x\).
- Suppose there are two men, Gabe and Al. There will be two contextual alternatives:
  - \(\{ (w, [x \rightarrow G, y \rightarrow b]) \mid G \text{ read } b \text{ in } w \}\)
- \{ (w, [x \rightarrow A, y \rightarrow b]) \mid A \text{ read } b \text{ in } w \}

- Familiarity is satisfied, since \( x \) and \( y \) are familiar throughout all states in the resulting context. Inquisitive closure introduces a new issue about whether \( x \) liked \( y \). Now we have four contextual alternatives:

  - \{ (w, [x \rightarrow G, y \rightarrow b]) \mid G \text{ read and liked } b \text{ in } w \}
  - \{ (w, [x \rightarrow G, y \rightarrow b]) \mid G \text{ read and didn’t like } b \text{ in } w \}
  - \{ (w, [x \rightarrow A, y \rightarrow b]) \mid A \text{ read and liked } b \text{ in } w \}
  - \{ (w, [x \rightarrow A, y \rightarrow b]) \mid A \text{ read and didn’t like } b \text{ in } w \}

### 4.5.2 Anaphora from polar questions

In dynamic inquisitive semantics, the possibilities of anaphora from polar questions fall straightforwardly under the witness generalization.

(30) A: Does Andreea have a\(^x\) husband? B: Yes, she’s married. He\(_x\)’s waiting outside.

(31) A: Does Andreea have a\(^x\) husband? B: ??No, she isn’t married. He\(_x\)’s waiting outside.

The question in the first conjunct has the following logical form:

(32) \( ?(\varepsilon_x \land H(x)) \)

It introduces two contextual alternatives (since negation is externally static):

- \{ (w, [x \rightarrow h]) \mid h \text{ is Andreea’s husband in } w \}
- \{ (w, []) \mid Andreea isn’t married in } w \}

If the alternative where Andreea isn’t married is contextually eliminated, then subsequent anaphora will be licensed (but only then).
4.5.3 Defects

A big problem for (Dotlačil & Roelofsen 2019) is anaphora in conjunctive questions:

(33) Did you read a\(^x\) paper, and was it\(_x\) interesting?

The reason is that the first conjunct introduces an alternative in which the \(x\) isn’t familiar.

This case is strikingly familiar to the following:

(34) John might have read a\(^x\) paper, and he might have found it\(_x\) interesting.

Indeed, without going into details (Dotlačil & Roelofsen 2019) suggest that (33) could be analyzed as a case of modal subordination (Roberts 1989).

Relatedly, since disjunction is internally static, (Dotlačil & Roelofsen 2019) have no chance of accounting for data like the following:

(35) Is there no\(^x\) bathroom or is it\(_x\) upstairs?
(36) a. Does Andreea not have a\(^x\) husband?
    b. No, she’s married - he\(_x\)’s waiting outside.

4.6 Intervention effects

Possible intervenors (Beck 2006):

- Focus sensitive operators: only, even…
- Quantifiers: every, no, most, few, always, never…
- Negation.

Note that - crucially - there is an anaphoric relation between \(\varepsilon_x\) and \(?_x\) in the formation of a question.

In order to capture intervention effects, we can assume that, in a language such as German, the surface position of the \textit{wh}-expression transparently reflects the scope of \(\varepsilon_x\).

In order to avoid the complexities of multiple \textit{wh}, we’ll consider an idealized case involving a single \textit{wh} in-situ.

\(^5\)On Wednesday, in my LFRG presentation, I’ll explore a different way of thinking about the projection/accessibility properties of sentences such as (34), by treating epistemic modals as filters in the sense of (Karttunen 1973). It’s possible that a similar strategy may be feasible for (33), but I leave this to future work.
Mary didn’t meet who?

\[-(x \land M(m, x)) \land ?_x\]

Since, \(-\) is externally static, \(?_x\) is unbound — given some auxiliary assumptions, this is assumed to lead to oddness. This parallels:

Mary didn’t meet anyone\(^x\). They\(_x\) are outside.

For similar reasons, this approach does reasonably well at accounting for intervention effects caused by quantificational determiners, with an important caveat.

Every boy met which who?

\[\forall_x (B(x) \rightarrow x \land M(x, y)) \land ?_x\]

As we learned in Filipe’s class however, quantifiers aren’t really externally static, so for this explanation to go through, we need to explain why \(?_x\) can’t pick up a discourse plural as in the following.

Every sophomore someone\(^x\) (different). They\(_x\) were all freshmen.

[Dotlačil & Roelofsen 2021b] mention another problem for the dynamic approach to intervention effects: focus sensitive particles don’t close off anaphoric information ([Haida 2007]).

This is illustrated by (44) - anaphora is possible, even when a paper takes scope below only (the most salient reading, i.e., John wrote a paper, and nobody else wrote any paper):

Only JOHN wrote a\(^x\) paper. It\(_x\) was long.

Even JOHN wrote a\(^x\) paper. It was long.

It might also initially seem tempting to apply the dynamic approach to intervention to intervention effects induced by disjunction in Japanese ([Shibata 2015]), given that disjunction is typically externally-static.

*Ken-ka Erika-ga nani-o yon-da no.
   Ken-or Erika-NOM what-ACC read-PST Q

Nani-o Ken-ka Erika-ga yon-da no.
What-ACC Ken-or Erika-NOM read-PST Q

A putative logical form for the intervention vs. non-intervention case:
This would fit quite nicely with the externally static disjunction of (Groenendijk & Stokhof 1991a), but - as we know - this is inadequate for natural language, and moreover (Dotlačil & Roelofsen 2019) explicitly adopt an externally dynamic disjunction in order to capture cases like the following (Stone 1992):

(49) Bill either rented a\textsuperscript{x} blue car or a\textsuperscript{x} red car. It\textsubscript{x} was probably a cabrolet.

In fact, the putative intervention LF is exactly a case parallel to (49), involving co-indexed indefinites in each disjunct.

5 Extension: inquisitive dynamic plural logic

(Dotlačil & Roelofsen 2021a) extend the basic semantics outlined in this section to a plural setting. Instead of taking possibilities to be world-assignment pairs, they take them to be pairings of worlds with plural assignments \((w, G)\).

This is illustrated below, for an information state post introduction of a dref \(x\):

\[
\begin{array}{c|ccc}
[x \rightarrow a] & w_a & w_{ab} & w_b & w_\emptyset \\
[x \rightarrow b] & \bullet & \bullet & \bullet & \bullet \\
[x \rightarrow a], [x \rightarrow b] & \bullet & \bullet & \bullet & \bullet \\
\end{array}
\]

Departing from (van den Berg 1996), (Dotlačil & Roelofsen 2021a) assume that atomic sentences are interpreted universally (see also (Champollion, Bledin & Li 2017)).

**Definition 5.1. Support** (plural assignments).

\[s \vdash P(x_1, \ldots, x_n) \iff \forall (w, G) \in s, \forall g \in G[g(x) \neq \ast \rightarrow (g(x_1), \ldots, g(x_n)) \in I_w(P)]\]

Plural random assignment needs to be defined as the cumulative generalization of random assignment, as Filipe discussed last week.

**Definition 5.2. Random assignment** (plural version).

\[G[x]H \iff \forall g \in G, \exists h \in H, g[x]h \land \forall h \in H, \exists g \in G, g[x]h\]
Given the expressivity of plural assignments, there are a number of different ways in which the witness identification operator $\varepsilon_x$ might be defined. (Dotlačil & Roelofsen 2021a) go for the following (quite complicated) definition:

**Definition 5.3. Witness identification operator** (plural).

- $C[\varepsilon_x] := \{ s[\varepsilon_x] \mid s \in C \}$

The intuition behind this is that $\varepsilon_x$ selects those states which contain enough information about the world to guarantee the existence of a unique value for $x$ globally.

If we apply $\varepsilon_x$ to our previous context, it gives rise to two contextual alternatives.

Note that resolving the issue raised by $\varepsilon_x$ does not necessarily uniquely specify the value of $G_x$. This derives the *mention some* reading of *wh*-questions.

(50) What is a famous french dish?

(51) $\varepsilon_x \land Fd(x) \land \varepsilon_x$

In order to derive the *mention-all* reading, (Dotlačil & Roelofsen 2021a) make use of a maximization operator.

**Definition 5.4. Maximization.** The maximization operator (relative to a variable $x$) keeps around states in $C$ which where the value collectively assigned to $x$ is maximal relative to the information about $x$ in $C$ globally.

- $C[\max_x] := \{ s \in C \mid \forall (w, G) \in s, \forall (w', G') \in \text{info}(C)[G'(x) \subseteq G(x)] \}$

Before applying $\varepsilon_x$, let’s consider the inquisitive context we’re dealing with:
Applying $?_x$ at $C$ will give us the states $s \in C$ which satisfy the following property: there is some individual $a$, s.t. every world in $s$ is paired with some plural assignment $G \in \text{info}_a(C)$ s.t. $G_x = \{a\}$.

Crucially any state containing $w_{ab}$ will fail to satisfy this property.

This derives a uniqueness inference:

(53) Which$^x$ linguist is giving a talk?

(54) $\varepsilon_x \land L(x) \land T(x) \land \max_x \land ?_x$

5.1 Q-subordination with questions

One possible motivation (not discussed by Dotlačil & Roelofsen 2021a):

(55) Which$^x$ book did each$^y$ boy read, and did they$^y$ each enjoy it$^x$?

References


